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Towards Aspect Invariant Feature Sets for Characterizing Three Dimensional Objects

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Abstract

The problem of characterizing three dimensional objects with a minimum number of feature sets is addressed. Previous research has been plagued by features which are a function of aspect angle and so efforts have concentrated on characterizing an object with several hundred prototype feature sets. In this work it is demonstrated that in many cases, the silhouette obtained from one view of the object can be derived from a linear transformation of the silhouette from another view. As a result of this relationship a single set of moments which is invariant to such a general linear transformation can be used to characterize many views of the same object and hence the number of prototype feature sets required to specify an object is reduced. In addition, it is demonstrated that for some objects it is advantageous to partition the object into regions in order to find the region of the object which is least dependent upon aspect angle.

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INTRODUCTION

For quite some time, the recognition of hand printed characters has received much attention and the methods developed have achieved significant success. In recent years, considerable interest has developed in the recognition of three dimensional objects. The research has been largely motivated by computer aided assembly and inspection for manufacturing and air traffic control problems. In the first case, the object is generally oriented so that recognition or inspection can be accomplished in two dimensions (1,3,4,6). Some attempts have been made to use multiple views of the object (2,5) but these methods are not applicable to air traffic control problems. In this case, information in three dimensions is necessary for recognition but only one view is available. This makes object recognition more difficult since parts of the object may be obscured by other parts of the object. Thus, the shape of the silhouette seen by the camera may be dependent upon the viewing or aspect angle. The orientation of the object may further alter the shape of the silhouette through rotation, translation, and size change (similitude).

From previous work in character recognition, it is well known that features, called moment invariants (9-11), can be used to describe the shape of a two dimensional image. They are invariant to rotation, translation, and similitude. These features have been directly applied to the recognition of three dimensional objects (12-16). In (12), Dudani extracts moment invariant features from aircraft shapes using over 500 different aspect angles for each aircraft type. For the six different type of aircraft used, the complete training sample set contained over 3000 live images. To classify an unknown aircraft, a modified K-nearest neighbor classifier was used to find the ten nearest neighbors among the 3000 element training set. A high degree of correct recognition was reported and is probably the most successful work to date. The major disadvantages of the method are the computational load, storage requirements, and the large number of images required to characterize an object. A similar aproach to recognizing ships was taken by Smith and Wright (13). Again, moments invariant to rotation, translation, and similitude were used to determine ship type. Here, the aspect problem was ingored by permitting only top views of ships to be admissible images. Another study for recognizing tanks (14, 17) was done and the methodology is the same as that of Dudani.

A comprehensive survey of automatic recognition of three dimensional objects using one optical sensor can be found in McGhee (15). Here, possible features are discussed for pattern recognition classifiers, such as the slope code formulation, Fourier descriptors (18), and moment invariants. In addition, some other, less successful, techniques for recognition of three dimensional objects are presented.

In all of the above methods for recognizing three dimensional objects, features are used which are a function of aspect angle and so characterizing an object typically requires several hundred feature sets. Such an approach does not consider the fact that some regions of the object are informationally richer than others (20). In this work, a region which is rich in information is one that has features which are aspect invariant while all other regions are considered to be ambiguous and are not useful for classifying the object. This idea is consistent with the conjecture that shape recognition is a hierarchical process (19).

In this work an attempt is made to characterize three dimensional objects with a minimum number of feature sets. Here it is demonstrated that for many cases, the silhouette obtained from one view of the object can be derived from a linear transformation of the silhouette from another view. As a result of this relationship, a set of moments which is invariant to such a general linear transformation can be used to characterize many views of the same object and hence the number of feature sets required to specify an object is reduced.

The next section discusses a set of features which are invariant under a general linear transformation. This is followed by some experiments with some common solid objects in which features are computed for various aspect angles. Next, aircraft shapes are considered and features are extracted for various regions of the aircraft at numerous aspect angles. Finally, suggestions for future research are presented.

Moments Invariant Under General Linear Tranformations

Let the two dimensional (p+q)th order moments of a discrete density distribution $f(x_i,y_i)$ be defined as

$$M_{pq} = \frac{1}{N} \sum_{i=1}^{N} (x_i-\bar{x}_i)^p (y_i-\bar{y}_i)^q f(x_i,y_i) \qquad (1)$$
$$p,q = 0,1,2...$$

where $\bar{x}$ is the mean of the coordinates x_i and $\bar{y}_i$ is the mean of the coordinate y_i. For a solid silhouette discretized to a matrix of zeros and ones the distribution $f(x_i,y_i) = 1$ for a point contained within the silhouette and zero otherwise. It can then be shown that moments of all orders exist and that the sequence $\{M_{pq}\}$ is uniquely determined by $f(x_i,y_i)$; and conversely, $f(x_i,y_i)$ is uniquely determined by $\{M_{pq}\}$, (11).

For the general linear transformation

$$\begin{bmatrix} u \\ v \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} \qquad (2)$$

there exists four absolute moment invariants given by

$$R_1 = (M_{20} M_{02} - M_{11}{}^2/ M_{00}{}^3 \qquad (3)$$

$$R_2 = ((M_{30} M_{03} - M_{21} M_{12})^2 - 4 (M_{30} M_{12} - M_{21}{}^2) (M_{21} M_{03} - M_{12}{}^2))/M_{00}{}^7 \qquad (4)$$

$$R_3 = (M_{20}(M_{21} M_{03} - M_{02}{}^2) - M_{11}(M_{30} M_{03} - M_{21} M_{12}) + M_{02}(M_{30} M_{12} - M_{21}{}^2))/ M_{00}{}^5 \qquad (5)$$

and

$$R_4 =(M_{30}{}^2 M_{02}{}^3 - 6M_{30} M_{21} M_{11} M_{02}{}^2 + 6 M_{30} M_{12} M_{02} (2 M_{11}{}^2 - M_{20} M_{02})$$
$$+ M_{30} M_{03} (6M_{20} M_{11} M_{02} - 8 M_{11}{}^3)$$
$$+ 9 M_{21}{}^2 M_{20} M_{02}{}^2 - 18 M_{21} M_{12} M_{20} M_{11} M_{02} + 6M_{21} M_{03} M_{20} (2M_{11}{}^2 - M_{20} M_{02})$$
$$+ 9 M_{12}{}^2 M_{20}{}^2 M_{02} - 6M_{12} M_{03} M_{11} M_{20}{}^2 + M_{03}{}^2 M_{20}{}^3)/M_{00}{}^7 \qquad (6)$$

Moments that are invariant to size change, rotation and translation have been used extensively for characterizing shape (11-16) and they are clearly a function of aspect angle (15). Moments that are invariant to a linear transformation can reduce this dependency on aspect angle. For example, consider a cube centered about the origin in an xyz coordinate system where ϕ represents rotation about the x axis, θ about the y axis and Ψ is rotation about the z axis. Initially, $\phi = \theta = \Psi = 0$. Rotating the object through any angle θ will not change the value of R_1, R_2, R_3, or R_4 since the silhouette obtained after rotation through θ given by the coordinates (u_i, v_i) can be obtained from the original silhouette by

$$\begin{bmatrix} u_i \\ v_i \end{bmatrix} = \begin{bmatrix} a & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x_i \\ y_i \end{bmatrix} \tag{7}$$

On the other hand, moments which are invariant to size change, rotation (this only includes rotation about an axis perpendicular to the original viewing plane) and translation will change after the rotation through θ. Consequently, moments invariant to a linear transformation are able to characterize more regions of the object. The net result is a reduction in the number of feature sets needed to characterize the object.

Investigating Aspect Invariance for Some Simple Objects

The features R_1, R_2, R_3 and R_4 were tested on a sphere, cube, cylinder and a cone. All images were binary images and were simulated on an IBM 370/158.

Let the object reside in a coordinate system xyz with the origin at or close to the center of gravity of the object. Initially, let the three axes xyz be colinear with the axes of the viewing or camera coordinate system, uvw. The viewing coordinate system is then rotated about the y axis through the angle θ , about the z axis through the angle Ψ and about the x axis through ϕ, in that order. The transformation which relates the two coordinate systems is

$$\begin{bmatrix} u \\ v \\ w \end{bmatrix} = \begin{bmatrix} \cos\theta\cos\Psi & \begin{matrix}\sin\theta\sin\phi \\ \cos\theta\sin\Psi\cos\phi\end{matrix} & \begin{matrix}\sin\theta\cos\phi + \\ \cos\theta\sin\Psi\sin\phi\end{matrix} \\ \sin\Psi & \cos\Psi\cos\phi & -\cos\Psi\sin\phi \\ -\sin\theta\cos\Psi & \begin{matrix}\cos\theta\sin\phi + \\ \sin\theta\sin\Psi\cos\phi\end{matrix} & \begin{matrix}\cos\theta\cos\phi - \\ \sin\theta\sin\Psi\sin\phi\end{matrix} \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} \tag{8}$$

The moments in equations (3)-(6) were then calculated from the solid silhouette in the uv plane for various orientations of the object. Now, the moments used here will be invariant to a rotation through an angle Ψ since this is just a special case of a linear transformation. Furthermore, for an image that possesses symmetry about a plane, the range of θ and ϕ necessary to cover all distinct views in smaller than the total 0 to 360 degree range (12). It can be shown that the significant range of values is

$$\begin{aligned} -90^\circ \leq \ & \theta \leq 90^\circ \\ 0^\circ \leq \ & \phi \leq 90^\circ \end{aligned} \tag{9}$$

All objects were represented in the uv plane by an 80 x 80 matrix of ones and zeros. In the original xyz coordinate system the sphere can be described by

$$x^2 + y^2 + z^2 \leq 100 \tag{10}$$

the cube can be described by the boundaries

$$x = \pm 5, \; y = \pm 5, \; z = \pm 5 \tag{11}$$

The cylinder has boundaries

$$x^2 + y^2 \leq 25, \; z = \pm 5 \tag{12}$$

and the cone can be described by

$$x^2 + y^2 \leq 25, \quad z \leq 5 - |x| \;, \quad z \leq 5 - |y| \tag{13}$$

A summary of the results is shown in Table 1 where $\bar{R}_i$ and σ_i^2 are the mean and variance respectively of R_i.

Although it is not evident from this table, R_4 was found to be very sensitive to noise or to the slightest distortion of the object. This is the reason for the large variance, σ_4^2. As a result of this observation, R_4 was not computed for the cube and cylinder. In addition, the features R_2 and R_3 are nearly zero in all cases except the cone. R_1 was the only moment found to be effective in measuring the dependency on aspect angle.

The quantity k refers to the number of feature vectors that are needed to characterize the object. This was calculated from the following algorithm.

1. Set k=1, Feat(j,1)=0 for j=1,2,3,4.

2. Compute R_j for j=1,2,3,4.

3. If
$$\frac{|R_1-\mathrm{Feat}(1,i)|+|R_2-\mathrm{Feat}(2,i)|+|R_3-\mathrm{Feat}(3,i)|+|R_4-\mathrm{Feat}(4,i)|}{|R_1|+|R_2|+|R_3|+|R_4|} \qquad (14)$$

   $< \alpha$ for any i=1,2..k then go to 2, otherwise proceed.

4. Set k=k+1, Feat(j,k) = R_j for j=1,2,3,4.

5. If all the orientations of the object have been considered then go to 6, otherwise go to 2.

6. Replace k by k-1.

In Table 1, α was chosen to be 0.05. Although there is no correlation between (14) and the error associated with a pattern classifier, it is still useful for comparisons between objects.

There is a considerable degree of similarity between the statistics of the features which implies that views of one object can be derived from a linear transformation of the view of another object. On the surface this looks discouraging but may actually prove to be of some value as discussed in the section on future research.

Investigation of Aircraft Shapes

Initially, the entire aircraft shown in figure A1 was rotated through the significant range of angles in order to calculate the moments R_1 through R_4 using program three in the appendix. This resulted in a large computational load and partial results indicated that the moments were widely different for nearly all aspect angles. Instead, the aircraft was partitioned into three regions: the wing, fuselage and tail as defined by the image coordinates in the appendix.

Each region of the aircraft was rotated through the significant range of angles and R_1 through R_4 were calculated. The results are summarized in Table 2.

Again, moments R_2 and R_3 are nearly zero for all orientations of the object. For the tail section, the variances of the moments are the smallest implying a relaxed dependency upon aspect angle. Also, the moments for the wing and fuselage are nearly the same in the case of R_1 and the large variance on R_4 in both cases leads one to conclude that these two regions would be difficult to separate in a pattern classifier with these moments as features. Evidently, the tail section would be the most useful for classifying aircraft type.

Summary

The use of moments which are invariant to a linear transformation were investigated for various objects and for various regions of an aircraft. Examining the statistics of the moments it appears that it would be difficult to classify certain objects using these moments as features because of their statistical similarity. On the other hand, the aircraft experiments seem to indicate that certain regions of the aircraft could be more easily classified than others.

Suggestions for Future Work

The original purpose of this work was to minimize the number of feature sets required to characterize an object, or a region of the object, with the intention that this reduced feature set could then be used for discriminating between classes of objects. It seems apparent from the results that many views of an object are just linear transformations of another view of the object and so moments can be used which result in a reduced feature set i.e. the aspect angle dependency has been relaxed. However, examining the values of R_1, R_2, R_3 and R_4 it is also apparent that they won't be able to separate classes with any significant degree of accuracy. On the other hand, there is an important conclusion which can be drawn from these data which is essentially a summary of the contribution of this investigation subject to further experimentation.

The computation of R_1 through R_4 seems to illustrate that (not only are many views of an object just linear transformations of another view of the object but that) a view of one object can be derived from a linear transformation of the view of another object. This is seen from the similarity between the statistics of R_1 through R_4 for different objects. This suggests that some regions are likely to cause problems in a pattern classifier if features are extracted from a silhouette that includes this region. Consequently, such objects or regions with statistically similar moments(which are invariant to a linear transformation) constitute ambiguous shapes. In the context of pattern recognition, the features derived from these ambiguous shapes occur near the sub-space boundaries and cause problems in a pattern recognition algorithm. As a result, shape algorithms are then, by definition, a collection of ad hoc rules needed to correct erroneous judgements caused by these so called difficult cases. This concept is similar to that taken by Blesser and Shillman (7,8) in their theory of character recognition in which ambiguously shaped characters formed the basis for their theory. With this concept of an ambiguous shape in mind it seems that further investigations with region analysis techniques (19) would determine whether they can aid in identifying these ambiguous shapes. Also, it has been established in a number of cases in character recognition that techniques such as polygonal approximation greatly reduce the influence of noise on the classification error (19), a problem which is significant when using moment invariants.

Actually what is needed in this area is a theory of shape based on human perception. The problem with any shape description lies in the fact that there exist ambiguous shapes. For example, how much shape distortion is permitted before a round shape becomes an oval shape? Indeed, it is these ambiguous shapes which show up as difficult cases in a pattern recognition algorithm. Consequently, the argument presented here is based on ambiguous shapes rather than archetyped shapes. Because objects differ markedly from the "norm" shape, a computer algorithm should measure the distance between the unknown shape and the boundary, rather than between the unknown and the norm shape. In effect, the boundary between classes is represented by ambiguous shapes. The problem here, is to find that boundary. To do so, a specific knowledge about human classification of shapes must first be obtained.

A knowledge about human classification of shapes can be found through psychological experimentation. First, define three classes of attributes that an image might have (7,8): physical attributes, perceptual attributes and functional attributes. Physical attributes are the parts of the image usually described in geometric or topological terms e.g. line, angles, areas, perimeters etc. Perceptual attributes are the qualities which are perceived , for example, two lines may be perceived as being equal even though they are physically unequal. Functional attributes provide the description of the image e.g. does a human subject classify the subject as being round or oval?

Consider the attribute height equals width (HEW) which might be used to classify an image as round or oval. The state of the physical attribute is determined by a physical measurement which mathematically answers the question, does height equal width? The state of the perceptual attribute is determined by experimentally answering the question, do you see height equal to width? The state of the functional attribute can only be determined by answering, is the image round or oval? For example, a truly circular image possesses HEW in all three cases: it is physically present, perceived as being present and also functions as being present since such an image would be labelled as round. Introducing a slight distortion in the circularity of the image (which would cause drastic changes in the moment invariants) , the height is not equal to the width so the physical attribute is not present. The perceptual attribute is present since they would be perceived as being equal and the functional attribute would be present since a human would label such an image as round. Introducing still more distortion in the circle so that it begins to approach an oval, the physical attribute is not present, the height is perceived as being unequal to the width, but yet it is unclear as to whether or not the HEW is functionally present since the image cannot be labelled either round or oval with any high degree of confidence. The image is therefore an ambiguous image due to the transitional state of the functional attribute HEW. When the image is clearly oval, all three attributes are not present and the image would be labelled as oval. The problem now is to find the mapping between the physical attributes and the functional attributes called Physical to Functional Rules (PFRs), (7,8).

The PFR for this example would involve finding a and b such that

$$b \leq \frac{H}{W} \leq a \qquad \text{Round}$$

$$\frac{H}{W} > a \qquad \text{oval}$$

$$\frac{H}{W} < b \qquad \text{oval}$$

Shillman {7,8} has developed several experimental methods for finding ambiguous characters. In our case, the ambiguous image and hence the PFR can be found through labeling experiments, reaction time experiments or goodness experiments.

In labeling experiments, subjects would label shapes along trajectories which go from one shape subspace to another. The boundary would then be identified as the shape or series of shapes which are assigned the two shape labels with equal probability.

In reaction time experiments, the amount of time required to label a shape can be measured. Near the boundary, subjects spend a greater than average amount of time identifying certain stimuli {7,8}. So reaction time is another technique for determining inter-shape boundary locations.

Goodness experiments are based on the concept of fuzzy sets. Subjects would be asked to rate each stimulus, using the integers 0-5, indicating how good a representation the stimulus is, for say, round or oval. It is argued that these concepts and experiments can be used successfully to develop a theory for describing shape.

These techniques can also contribute to developing a theory for recognizing images i.e. separating an image from the background. Numerous edge detection algorithm exist, but none are based on human perception. Threshold selection techniques have been used as a basic tool in image segmentation, but little work has been done on the problem of evaluating a threshold of an image. Some authors evaluate thresholds based on a busyness criterion and a descrepancy criterion. However, what proof is there that these two criteria have any significance in terms of human perception? The concepts presented here could easily develop into a theory for thresholding image boundaries.

In a similar sense, humans have the ability to recognize and classify objects in the prescence of noise. Noisy images have caused tremendous problems in pattern recognition based system. However, the experiments described earlier could just as easily have been carried out with noisy images. Subjects, can be shown a trajectory of computer processed shapes that have been corrupted by noise. Labeling experiments would then automatically aid in classifying the noisy shape. This would then lead to the selection of a and b in the noisy case.

Another major advantage of the procedure described here is that it lends itself to the design of a decision tree classifier.

Finding a and b amounts to finding a threshold at a decision point in the tree.

In this section it has been argued that theories for image recognition and description can be developed based on human perception. The net result would remove the ad hoc approach that presently dominates this area.

Table 1

Object	Sphere	Cube	Cylinder	Cone
$\bar{R}_1$ σ_1^2	2.193 $1.95\ 10^{-4}$	1.041 0.015	1.109 0.068	0.482 $9.16\ 10^{-3}$
$\bar{R}_2$ σ_2^2	$-1.28\ 10^{-7}$ $3.75\ 10^{-13}$	0.000 0.000	$8.24\ 10^{-7}$ $2.23\ 10^{-10}$	$-1.92\ 10^{-3}$ $1.87\ 10^{-5}$
$\bar{R}_3$ σ_3^2	$-2.11\ 10^{-4}$ $2.37\ 10^{-7}$	0.000 0.000	$5.5\ 10^{-4}$ $1.04\ 10^{-6}$	-0.0144 $6.86\ 10^{-4}$
$\bar{R}_4$ σ_4^2	1.292 8.961	- -	- -	1.761 7.801
k	1	6	7	38

Table 2

Region	Wing	Tail	Fuselage
$\bar{R}_1$	1.14	0.834	1.27
σ_1^2	0.24	0.056	0.35
$\bar{R}_2$	$-4.55\ 10^{-5}$	$-4.95\ 10^{-5}$	$-8.98\ 10^{-4}$
σ_2^2	$8.29\ 10^{-9}$	$8.47\ 10^{-9}$	$2.69\ 10^{-6}$
$\bar{R}_3$	$-7.32\ 10^{-3}$	$-4.1\ 10^{-3}$	$-3.06\ 10^{-2}$
σ_3^2	$2.94\ 10^{-5}$	$2.03\ 10^{-5}$	$1.12\ 10^{-3}$
$\bar{R}_4$	15.6	1.96	107.00
σ_4^2	266.0	6.09	$1.76\ 10^{4}$

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Appendix 1

This appendix contains the programs and image coordinates used to calculate the moments R_j for j=1,2..4.

The first program calculates the moments for the sphere described in a previous section as well as the mean and variance of these moments. With minor modifications the program was also used to calculate moments for a cube, cylinder and cone.

Program 2 was used to rotate various regions of the aircraft shapes shown in figure A1 through the significant range of angles while calculating the moments from the solid silhouette. The last statement in the program determines what data files are used and hence what region is under investigation. The data files of the image coordinates are in the latter part of the appendix.

Next is figure A1 which contains the ortogonal views of a typical aircraft. The boundaries of these views were discretized to form the data files of image coordinates.

Program 3 was used to rotate the entire aircraft and to calculate the moments of the solid silhpuette. This program includes boundary tracing algorithms which had to be written to circumvent the complexities associated with certain regions of the orthogonal views e.g. concavities. Options are also included for working with the solid silhouette or just the boundary of the silhouette in the uv plane. A smoothing algorithm is also provided to smooth the boundary before calculating the moments.

Data set 1 contains the image coordinates obtained from the views of the aircraft in figure A1. The x axis corresponds to the roll axis with the nose in the positive x direction. The y axis corresponds to the pitch axis of the aircraft with the left wing on the positive y axis. The z axis is the yaw axis with the tail on the positive z axis.

Data sets 2,3 and 4 are the image boundary coordinates used for the regions of the airplane designated as wing (2), fuselage (3) and tail (4).

Program 1

```
      DIMENSION XY(100,100),FEAT(4,100),XMEAN(4),VAR(4),FT(4,200)
C
      DELTAX=.5
      ALPHA=.05
      DEGINC=10.
      NDEG=360./DEGINC
      DEGINC=DEGINC*2.*3.14159/360.
C
C
      HNXYZ=10.
      NPTS=40
      HNPTS=NPTS/2
      HNXYZS=HNXYZ**2
      INC=10
      STEP=DELTAX**2
      NXYZ=2*(HNXYZ-INC*STEP)/DELTAX+2.*INC+2
      IDIFF=NXYZ-INC
      START=-HNXYZ-STEP

C
      DO 50 I=1,4
      XMEAN(I)=0.
     VAR(I)=0.
   50 FEAT(I,1)=0.
       K195=0
      K=1
C
      PSI=0.
      THETA=-DEGINC-3.14159*.5
      CSPS=COS(PSI)
      SNPS=SIN(PSI)
C
      DO 800 K2=1,19
      THETA=THETA+DEGINC
      PHI=-DEGINC
      CSTH=COS(THETA)
      SNTH=SIN(THETA)
      C1=CSTH*CSPS
C
      DO 800 K3=1,10
      PHI=PHI+DEGINC
      CSPH=COS(PHI)
      SNPH=SIN(PHI)
C
      C2=SNTH*SNPH-CSTH*SNPS*CSPH
      C3=CSTH*SNPS*SNPH+SNTH*CSPH
      C4=CSPS*CSPH
      C5=CSPS*SNPH
C
      DO 10 I4=1,NPTS
      DO 10 I5=1,NPTS
   10 XY(I4,I5)=0.
C
```

```
C
      X=START
      DO 600 I=1,NXYZ
      IF(I.LT.INC.OR.I.GT.IDIFF) GO TO 601
      X=X+DELTAX
      GO TO 602
 601     X=X+STEP
 602  Y=START
      XX=X*X
      DO 600 J=1,NXYZ
      IF(J.LT.INC.OR.J.GT.IDIFF) GO TO 603
      Y=Y+DELTAX
      GO TO 604
 603    Y=Y+STEP
 604  Z=START
      YY=Y*Y
      DO 600 K7=1,NXYZ
      IF(K7.LT.INC.OR.K7.GT.IDIFF) GO TO 605
      Z=Z+DELTAX
      GO TO 606
 605   Z=Z+STEP
 606    CONTINUE
      IF(XX+YY+Z*Z.GT.HNXYZS) GO TO 600
C
      XP=C1*X+C2*Y+C3*Z+HNPTS
      YP=SNPS*X+C4*Y-C5*Z+HNPTS
      I1=XP
      I2=YP
      XP1=I1
      YP1=I2
      IF(XP-XP1.GT.0.5) I1=I1+1
      IF(YP-YP1.GT.0.5) I2=I2+1
C
      XY(I1,I2)=1.
 600  CONTINUE
C
C      COMPUTE MOMENTS
C
      U=0.
      UM10=0.
      UM01=0.
C
      DO 700 I1=1,NPTS
      DO 700 I2=1,NPTS
      IF(XY(I1,I2).EQ.0.) GO TO 700
      U=U+1.
      UM10=UM10-I1+HNPTS
      UM01=UM01+I2-HNPTS
 700   CONTINUE
```

```
C
      XBAR=UM10/U
      YBAR=UM01/U
C
      AA=0.
      BB=0.
      CC=0.
      A=0.
      B=0.
      C=0.
      D=0.
C
      DO 750 I1=1,NPTS
      DO 750 I2=1,NPTS
      IF(ABS(XY(I1,I2)).LT..1) GO TO 750
      TEMPX=-I1+HNPTS-XBAR
      TEMPY=I2-HNPTS-YBAR
      TEMP1=TEMPX**2
      AA=AA+TEMP1
C
      BB=BB+TEMPX*TEMPY
      TEMP2=(TEMPY)**2
      CC=CC+TEMP2
C
      A=A+(TEMPX)*TEMP1
      B=B+(TEMPY)*TEMP1
      C=C+(TEMPX)*TEMP2
       D=D+(TEMPY)*TEMP2
 750  CONTINUE
C
        U2=U*U
      U3=U2*U
C
      R1=(AA*CC-BB*BB)/U3
      R2=((A*D-B*C)**2)-4.*(A*C-B*B)*(B*D-C*C)
      R2=R2/(U3*U2*U2)
      R3=AA*(B*D-C*C)-BB*(A*D-B*C)+CC*(A*C-B*B)
      R3=R3/(U3*U2)
      TEMP=2.*BB*BB-AA*CC
      ABC=AA*BB*CC
      CC2=CC*CC
      CC3=CC2*CC
      AA2=AA*AA
      AA3=AA2*AA
C
      R4=A*A*CC3-6.*A*B*BB*CC2+6.*A*C*CC*TEMP
      R4=R4+A*D*(6.*ABC-8.*BB*BB*BB)+9.*B*B*AA*CC2-18.*B*C*ABC
C
      R4=R4+6.*B*D*AA*TEMP+9.*C*C*AA2*CC
      R4=R4-6.*C*D*BB*AA2+D*D*AA3
      R4=R4/(U3*U2*U2)
```

```
C
      K195=K195+1
      FT(1,K195)=R1
      FT(2,K195)=R2
      FT(3,K195)=R3
      FT(4,K195)=R4
C
C
      TOT=ABS(R1)+ABS(R2)+ABS(R3)+ABS(R4)
      DO 776 K9=1,K
C
      IF((ABS(R1-FEAT(1,K9))+ABS(R2-FEAT(2,K9))+ABS(R3-FEAT(3,K9))+
     1ABS(R4-FEAT(4,K9)))/TOT.LT.ALPHA) GO TO 790
 776   CONTINUE
      K=K+1
      FEAT(1,K)=R1
      FEAT(2,K)=R2
      FEAT(3,K)=R3
      FEAT(4,K)=R4
 790  WRITE(6,200) R1,R2,R3,R4,K,THETA,PSI,PHI
 200  FORMAT(E10.3,2X,E10.3,2X,E10.3,2X,E10.3,I5,3F10.4)
 800  CONTINUE
C
C      COMPUTE THE MEAN AND VARIANCE OF THE FEATURES
C
      DO 810 I=1,K195
      DO 810 J=1,4
  810  XMEAN(J)=XMEAN(J)+FT(J,I)
      DO 820 J=1,4
  820 XMEAN(J)=XMEAN(J)/K195
      DO 830 I=1,K195
      DO 830 J=1,4
  830 VAR(J)=VAR(J)+(FT(J,I)-XMEAN(J))**2
      DO 840 I=1,4
  840 VAR(I)=VAR(I)/(K195-1)
      WRITE(6,200) (XMEAN(K),K=1,4)
      WRITE(6,200) (VAR(K),K=1,4)
C
      STOP
      END
```

Program 2

```
      DIMENSION XXY(80,80),FEAT(4,100),XY(200,2),XZ(200,2),
     1YZ(200,2),XMEAN(4),VAR(4),FT(4,200)
C
      SCF=.5
C
C       READ NUMBER OF BOUNDARY POINTS
C
      READ(5,35) NBXY
   35 FORMAT(I3)
C
      DO 40 I=1,NBXY
      READ(5,45) XY(I,1),XY(I,2)
       XY(I,1)=SCF*XY(I,1)
      XY(I,2)=SCF*XY(I,2)
   40 CONTINUE
   45 FORMAT(F5.0,F8.0)
C
      READ(5,35) NBXZ
      DO 41 I=1,NBXZ
      READ (5,45) XZ(I,1),XZ(I,2)
      XZ(I,1)=SCF*XZ(I,1)
      XZ(I,2)=SCF*XZ(I,2)
  41  CONTINUE
C
      READ(5,35) NBYZ
      DO 42 I=1,NBYZ
      READ(5,45) YZ(I,1),YZ(I,2)
      YZ(I,1)=SCF*YZ(I,1)
      YZ(I,2)=SCF*YZ(I,2)
   42 CONTINUE
C
C
      ALPHA=.05
      DEGINC=10.
      DEGINC=DEGINC*3.14159/180.
      DO 50 I=1,4
      XMEAN(I)=0.
      VAR(I)=0.
   50 FEAT(I,1)=0.
      K=1
      K195=0
C
      PSI=0.
      THETA=-DEGINC-3.14159/2.
      CSPS=COS(PSI)
      SNPS=SIN(PSI)
      FVW=80.
      FVWD2=-FVW/2.
      N=80
      RN=N
      HNPTS=RN/2.
      DELTA=FVW/RN
```

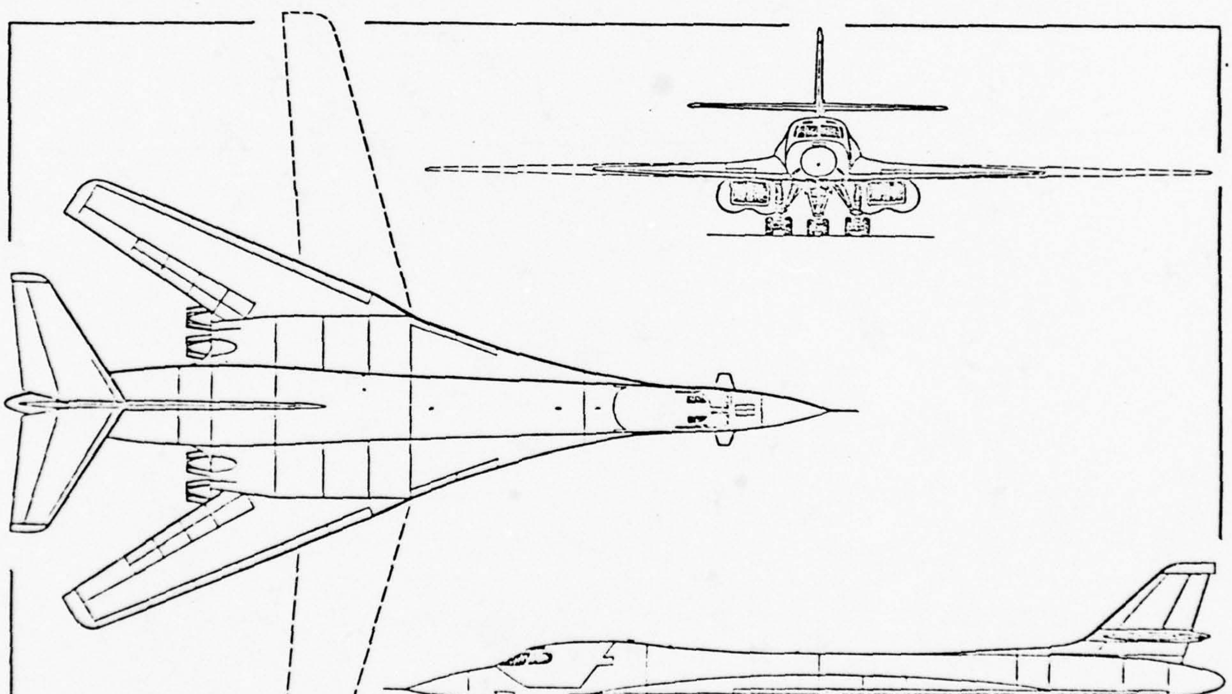
```
C
C     SEARCH FOR MAX AND MIN VALUES OF Z
C
      ZMAX=FVWD2
      ZMIN=-ZMAX
      XMAX=ZMAX
      XMIN=ZMIN
      YMAX=ZMAX
      YMIN=ZMIN
      DO 10 I=1,NBXZ
      TESTZ=XZ(I,2)
      TESTX=XZ(I,1)
      IF(TESTZ.GT.ZMAX) ZMAX=TESTZ
      IF(TESTZ.LT.ZMIN) ZMIN=TESTZ
      IF(TESTX.GT.XMAX) XMAX=TESTX
      IF(TESTX.LT.XMIN) XMIN=TESTX
   10 CONTINUE
      M=ABS(ZMAX-ZMIN)/DELTA+1.
      MM=ABS(XMAX-XMIN)/DELTA+1.
C
      DO 15 I=1,NBXY
      TESTY=XY(I,2)
      IF(TESTY.GT.YMAX) YMAX=TESTY
      IF(TESTY.LT.YMIN) YMIN=TESTY
   15  CONTINUE
      MMM=ABS(YMAX-YMIN)/DELTA+1.
C
C
      WRITE(6,211) NBXY,NBXZ,NBYZ,N
  211 FORMAT(2X,'NBXY=',I5,'NBXZ=',I5,'NBYZ=',I5,'ARRAY SIZE=',I5)
C
      DO 800 K2=1,19
      THETA=THETA+DEGINC
      PHI=-DEGINC
      CSTH=COS(THETA)
      SNTH=SIN(THETA)
      C1=CSTH*CSPS
C
      DO 800 K3=1,10
      PHI=PHI+DEGINC
      CSPH=COS(PHI)
      SNPH=SIN(PHI)
C
      C2=SNTH*SNPH-CSTH*SNPS*CSPH
      C3=CSTH*SNPS*SNPH+SNTH*CSPH
      C4=CSPS*CSPH
      C5=CSPS*SNPH
C
      DO 5 I=1,N
      DO 5 J=1,N
    5 XXY(I,J)=0.
C
      Z=ZMIN-DELTA
      DO 600 I=1,M
      Z=Z+DELTA
      Y=YMIN-DELTA
      DO 600 II=1,MMM
      Y=Y+DELTA
      X=XMIN-DELTA
      DO 600 III=1,MM
      X=X+DELTA
```

```
C
C      CHECK THE SILHOUETTE IN THE XY PLANE
C
      L1=0
      L2=0
      L4=0
      L5=0
      DO 330 I1=1,NBXY
      IF(ABS(XY(I1,1)-X).GT.0.001) GO TO 329
      TEMP=XY(I1,2)
      IF(Y.GE.TEMP) L1=1
      IF(Y.LE.TEMP) L2=1
      L3=L1+L2
  329 CONTINUE
      IF(ABS(XY(I1,2)-Y).GT.0.001) GO TO 331
      TEMP=XY(I1,1)
      IF(X.GE.TEMP) L4=1
      IF(X.LE.TEMP) L5=1
      L6=L4+L5
  331 IF(L3.EQ.2.AND.L6.EQ.2) GO TO 335
  330 CONTINUE
      GO TO 600
 335   CONTINUE
C      CHECK THE SILHOUETTE IN THE YZ PLANE
      L1=0
      L2=0
      L4=0
      L5=0
      DO 430 I1=1,NBYZ
      IF(ABS(YZ(I1,1)-Y).GT.0.001) GO TO 429
      TEMP=YZ(I1,2)
      IF(Z.GE.TEMP) L1=1
      IF(Z.LE.TEMP) L2=1
      L3=L1+L2
  429 CONTINUE
      IF(ABS(YZ(I1,2)-Z).GT.0.001) GO TO 431
      TEMP=YZ(I1,1)
      IF(Y.GE.TEMP)L4=1
      IF(Y.LE.TEMP)L5=1
      L6=L4+L5
  431 IF(L3.EQ.2.AND.L6.EQ.2) GO TO 435
  430 CONTINUE
      GO TO 600
C      CHECK THE XZ PLANE
435   L1=0
      L2=0
     L4=0
      L5=0
      DO 440 I1=1,NBXZ
      IF(ABS(XZ(I1,1)-X).GT.0.001) GO TO 439
      TEMP=XZ(I1,2)
      IF(Z.GE.TEMP)  L1=1
      IF(Z.LE.TEMP) L2=1
      L3=L1+L2
  439 CONTINUE
      IF(ABS(XZ(I1,2)-Z).GT.0.001) GO TO 441
      TEMP=XZ(I1,1)
      IF(X.GE.TEMP) L4=1
      IF(X.LE.TEMP) L5=1
      L6=L4+L5
  441 IF(L3.EQ.2.AND.L6.EQ.2) GO TO 14
  440 CONTINUE
```

```
C
      GO TO 600
C
C
   14 XP=-(C1*X+C2*Y+C3*Z)+HNPTS
      YP=SNPS*X+C4*Y-C5*Z+HNPTS
      I1=XP
      I2=YP
C
      XXY(I1,I2)=1.
C
C
  600 CONTINUE
C
C      COMPUTE MOMENTS
C
      U=0.
      UM10=0.
      UM01=0.
C
      DO 771 I1=1,N
      DO 771 I2=1,N
      IF(ABS(XXY(I1,I2)).LT.0.01) GO TO 771
      U=U+1.
      UM10=UM10-I1+HNPTS
      UM01=UM01+I2-HNPTS
  771 CONTINUE
C
      XBAR=UM10/U
      YBAR=UM01/U
C
      AA=0.
      BB=0.
      CC=0.
      A=0.
      B=0.
      C=0.
      D=0.
C
      DO 750 I1=1,N
      DO 750 I2=1,N
      IF(ABS(XXY(I1,I2)).LT.0.01) GO TO 750
      TEMPX=-I1+HNPTS-XBAR
      TEMPY=I2-HNPTS-YBAR
      TEMP1=TEMPX**2
      AA=AA+TEMP1
C
      BB=BB+TEMPX*TEMPY
      TEMP2=TEMPY**2
      CC=CC+TEMP2
C
      A=A+TEMPX*TEMP1
      B=B+TEMPY*TEMP1
      C=C+TEMPX*TEMP2
      D=D+TEMPY*TEMP2
  750 CONTINUE
```

```
C
      U2=U*U
      U3=U2*U
C
      R1=(AA*CC-BB*BB)/U3
      R2=((A*D-B*C)**2)-4.*(A*C-B*B)*(B*D-C*C)
      R2=R2/(U3*U2*U2)
      R3=AA*(B*D-C*C)-BB*(A*D-B*C)+CC*(A*C-B*B)
      R3=R3/(U3*U2)
      TEMP=2.*BB*BB-AA*CC
      ABC=AA*BB*CC
      CC2=CC*CC
      CC3=CC2*CC
      AA2=AA*AA
      AA3=AA2*AA
C
      R4=A*A*CC3-6.*A*B*BB*CC2+6.*A*C*CC*TEMP
      R4=R4+A*D*(6.*ABC-8.*BB*BB*BB)+9.*B*B*AA*CC2-18.*B*C*ABC
C
      R4=R4+6.*B*D*AA*TEMP+9.*C*C*AA2*CC
      R4=R4-6.*C*D*BB*AA2+D*D*AA3
      R4=R4/(U3*U2*U2)
C
      K195=K195+1
      FT(1,K195)=R1
      FT(2,K195)=R2
      FT(3,K195)=R3
      FT(4,K195)=R4
      TOT=ABS(R1)+ABS(R2)+ABS(R3)+ABS(R4)
      DO 776 K9=1,K
      IF((ABS(R1-FEAT(1,K9))+ABS(R2-FEAT(2,K9))+ABS(R3-FEAT(3,K9))+
     1ABS(R4-FEAT(4,K9)))/TOT.LT.ALPHA) GO TO 790
  776 CONTINUE
C
      K=K+1
      FEAT(1,K)=R1
      FEAT(2,K)=R2
      FEAT(3,K)=R3
       FEAT(4,K)=R4
C
  790 WRITE(6,200) R1,R2,R3,R4,K,THETA,PSI,PHI
  200 FORMAT(E10.3,2X,E10.3,2X,E10.3,2X,E10.3,I5,3F10.4)
  800 CONTINUE
```

```
C
C       COMPUTE THE MEAN AND VARIANCE OF FEATURES
C
      DO 810 I=1,K195
      DO 810 J=1,4
  810  XMEAN(J)=XMEAN(J)+FT(J,I)
      DO 820 J=1,4
  820 XMEAN(J)=XMEAN(J)/K195
      DO 830 I=1,K195
      DO 830 J=1,4
  830  VAR(J)=VAR(J)+(FT(J,I)-XMEAN(J))**2
      DO 840 I=1,4
  840  VAR(I)=VAR(I)/(K195-1)
      WRITE(6,200) (XMEAN(K),K=1,4)
      WRITE(6,200) (VAR(K),K=1,4)
C
 500  STOP
      END
/DATA
/INCLUDE FUSEXY,FUSEXZ,FUSEYZ
```



North American Rockwell B-1 Strategic Bomber

Figure A1

Program 3

```
      DIMENSION AXY(100,100),FEAT(4,100),B8(8),XY(400,2),XZ(400,2),
     1YZ(400,2),XYS(400,2)
C
C       IFLAG .NE. 1 PRINTS OUT THE SOLID SILHOUETTE
C       JFLAG .NE. 1 SMOOTHS THE BOUNDARY
C       KFLAG .NE. 1 PRINTS OUT THE SILHOUETTE BOUNDARY
C
      IFLAG=1
      JFLAG=1
      KFLAG=1
      SCF=.5
C
C       READ NUMBER OF BOUNDARY POINTS
C
      READ(5,35) NBXY
   35 FORMAT(I3)
C
      DO 40 I=1,NBXY
      READ(5,45) XY(I,1),XY(I,2)
       XY(I,1)=SCF*XY(I,1)
      XYS(I,1)=XY(I,1)
      XY(I,2)=SCF*XY(I,2)
      XYS(I,2)=XY(I,2)
   40 CONTINUE
   45 FORMAT(F5.0,F8.0)
C
      READ(5,35) NBXZ
      DO 41 I=1,NBXZ
      READ (5,45) XZ(I,1),XZ(I,2)
      XZ(I,1)=SCF*XZ(I,1)
      XZ(I,2)=SCF*XZ(I,2)
   41 CONTINUE
C
      READ(5,35) NBYZ
      DO 42 I=1,NBYZ
      READ(5,45) YZ(I,1),YZ(I,2)
      YZ(I,1)=SCF*YZ(I,1)
      YZ(I,2)=SCF*YZ(I,2)
   42 CONTINUE
C
C
      ALPHA=.05
      DEGINC=10.
      NDEG=360./DEGINC
      DEGINC=DEGINC*3.14159/180.
      DO 50 I=1,4
   50 FEAT(I,1)=0.
      K=1
C
      PSI=0.
      THETA=-DEGINC-3.14159/2.
      CSPS=COS(PSI)
      SNPS=SIN(PSI)
```

```
      FVW=80.
      FVWD2=-FVW/2.
      N=80
      RN=N
      HNPTS=RN/2.
      DELTA=FVW/RN
C
C     SEARCH FOR MAX AND MIN VALUES OF Z
C
      ZMAX=FVWD2
      ZMIN=-ZMAX
      DO 10 I=1,NBXZ
      TESTZ=XZ(I,2)
      IF(TESTZ.GT.ZMAX) ZMAX=TESTZ
      IF(TESTZ.LT.ZMIN) ZMIN=TESTZ
   10 CONTINUE
      M=ABS(ZMAX-ZMIN)/DELTA+1.
C
      WRITE(6,211) NBXY,NBXZ,NBYZ,N
  211 FORMAT(2X,'NBXY=',I5,'NBXZ=',I5,'NBYZ=',I5,'ARRAY SIZE=',I5)
C
      DO 800 K2=1,19
      THETA=THETA+DEGINC
      PHI=-DEGINC
      CSTH=COS(THETA)
      SNTH=SIN(THETA)
      C1=CSTH*CSPS
C
      DO 800 K3=1,10
      PHI=PHI+DEGINC
      CSPH=COS(PHI)
      SNPH=SIN(PHI)
C
      C2=SNTH*SNPH-CSTH*SNPS*CSPH
      C3=CSTH*SNPS*SNPH+SNTH*CSPH
      C4=CSPS*CSPH
      C5=CSPS*SNPH
C
      DO 5 I=1,N
      DO 5 J=1,N
    5 AXY(I,J)=0.
C
      DO 600 I=1,NBXY
      X=XY(I,1)
      IF(X.EQ.10000.) GO TO 600
      JJ=I+1
      YINIT=XY(I,2)
      IF(XY(JJ,1).EQ.10000.) GO TO 615
      DO 610 J=JJ,NBXY
      IF(X.EQ.XY(J,1)) GO TO 620
  610 CONTINUE
```

```
  615  Y=YINIT
       YFIN=YINIT
       XY(I,1)=10000.
       GO TO 427
  620  YFIN=XY(J,2)
       Y=YINIT-DELTA
       IF(YFIN.LT.YINIT) Y=YINIT+DELTA
       XY(J,1)=10000.
  425  CONTINUE
       IF(YFIN.EQ.YINIT) GO TO 600
       IF(YFIN.LT.YINIT) GO TO 426
       Y=Y+DELTA
       IF(Y.GT.YFIN) GO TO 600
       GO TO 427
  426  Y=Y-DELTA
       IF(Y.LT.YFIN) GO TO 600
  427   Z=ZMIN-DELTA
       DO 450 K39=1,M
       Z=Z+DELTA
C        CHECK THE SILHOUETTE IN THE YZ PLANE
       L1=0
       L2=0
       DO 430 I1=1,NBYZ
       IF(ABS(YZ(I1,1)-Y).GT.0.001) GO TO 430
       TEMP=YZ(I1,2)
       IF(Z.GE.TEMP) L1=1
       IF(Z.LE.TEMP) L2=1
       L3=L1+L2
       IF(L3.EQ.2) GO TO 435
  430  CONTINUE
       GO TO 450
C        CHECK THE XZ PLANE
435    L1=0
       L2=0
       DO 440 I1=1,NBXZ
       IF(ABS(XZ(I1,1)-X).GT.0.001) GO TO 440
       TEMP=XZ(I1,2)
       IF(Z.GE.TEMP)  L1=1
       IF(Z.LE.TEMP) L2=1
       L3=L1+L2
       IF(L3.EQ.2) GO TO 14
  440  CONTINUE
C
       GO TO 450
C
```

```
C
   14  XP=(C1*X+C2*Y+C3*Z)+HNPTS
       YP=SNPS*X+C4*Y-C5*Z+HNPTS
       I1=XP
       I2=YP
C
       AXY(I1,I2)=1.
C
C
  450  CONTINUE
       GO TO 425
  600  CONTINUE
C
C        COMPUTE MOMENTS
C
       U=0.
       UM10=0.
       UM01=0.
C
       DO 771 I1=1,N
       DO 771 I2=1,N
       IF(AXY(I1,I2).EQ.0.) GO TO 771
       U=U+1.
       X=I1-HNPTS
       UM10=UM10+X
       Y=I2-HNPTS
       UM01=UM01+Y
  771  CONTINUE
C
       XBAR=UM10/U
       YBAR=UM01/U
C
       AA=0.
       BB=0.
       CC=0.
       A=0.
       B=0.
       C=0.
       D=0.
C
       DO 750 I1=1,N
       DO 750 I2=1,N
       IF(AXY(I1,I2).EQ.0.) GO TO 750
       X=I1-HNPTS
       Y=I2-HNPTS
       TEMPX=X-XBAR
       TEMPY=Y-YBAR
       TEMP1=TEMPX**2
       AA=AA+TEMP1
C
       BB=BB+TEMPX*TEMPY
       TEMP2=TEMPY**2
       CC=CC+TEMP2
```

```
C
      A=A+TEMPX*TEMP1
      B=B+TEMPY*TEMP1
      C=C+TEMPX*TEMP2
      D=D+TEMPY*TEMP2
 750  CONTINUE
C
      U2=U*U
      U3=U2*U
C
      R1=(AA*CC-BB*BB)/U3
      R2=((A*D-B*C)**2)-4.*(A*C-B*B)*(B*D-C*C)
      R2=R2/(U3*U2*U2)
      R3=AA*(B*D-C*C)-BB*(A*D-B*C)+CC*(A*C-B*B)
      R3=R3/(U3*U2)
      TEMP=2.*BB*BB-AA*CC
      ABC=AA*BB*CC
      CC2=CC*CC
      AA2=AA*AA
      AA3=AA2*AA
C
      R4=A*A*CC3-6.*A*B*BB*CC2+6.*A*C*CC*TEMP
      R4=R4+A*D*(6.*ABC-8.*BB*BB*BB)+9.*B*B*AA*CC2-18.*B*C*ABC
C
      R4=R4+6.*B*D*AA*TEMP+9.*C*C*AA2*CC
      R4=R4-6.*C*D*BB*AA2+D*D*AA3
      R4=R4/(U3*U2*U2)
C
      TOT=ABS(R1)+ABS(R2)+ABS(R3)+ABS(R4)
      DO 776 K9=1,K
      IF((ABS(R1-FEAT(1,K9))+ABS(R2-FEAT(2,K9))+ABS(R3-FEAT(3,K9))+
     1ABS(R4-FEAT(4,K9)))/TOT.LT.ALPHA) GO TO 790
 776  CONTINUE
C
      K=K+1
      FEAT(1,K)=R1
      FEAT(2,K)=R2
      FEAT(3,K)=R3
       FEAT(4,K)=R4
C
      DO 788 I1=1,NBXY
      XY(I1,1)=XYS(I1,1)
 788  XY(I1,2)=XYS(I1,2)
C
 790  WRITE(6,200) R1,R2,R3,R4,K,THETA,PSI,PHI
 200  FORMAT(E10.3,2X,E10.3,2X,E10.3,2X,E10.3,I5,3F10.4)
 800  CONTINUE
C
      IF(IFLAG.EQ.1) GO TO 61
```

```
C
      DO 60 I=1,N
   60 WRITE(6,200) (AXY(I,J),J=1,N)
  61  CONTINUE
C
      IF(JFLAG.EQ.1) GO TO 701
C      SMOOTH THE BOUNDARY POINTS
      NN=N-2
C
      DO 700 I=1,NN
      DO 700 J=1,NN
      IF(AXY(I,J).EQ.BLANK) GO TO 700
      IF(AXY(I+1,J).EQ.SYMBOL) GO TO 700
      IF(AXY(I+1,J+1).EQ.SYMBOL) GO TO 700
      IF(AXY(I,J+1).EQ.SYMBOL) GO TO 700
      J1=0
      J2=0
      J3=0
      J4=0
      J5=0
      IF(AXY(I+2,J).EQ.SYMBOL) J1=1
      IF(AXY(I+2,J+1).EQ.SYMBOL) J2=1
      IF(AXY(I+2,J+2).EQ.SYMBOL) J3=1
      IF(AXY(I+1,J+2).EQ.SYMBOL) J4=1
      IF(AXY(I,J+2).EQ.SYMBOL) J5=1
      JSUM=J1+J2+J3+J4+J5
      IF(J3.EQ.1.AND.J4.EQ.1) AXY(I+1,J+1)=SYMBOL
      IF(J2.EQ.1.AND.J3.EQ.1) AXY(I+1,J+1)=SYMBOL
      IF(JSUM.EQ.0) GO TO 700
      IF(JSUM.GT.1) GO TO 700
      JUMP=J1*1+J2*1+J3*2+J4*2+J5*3
      GO TO (1,2,3),JUMP
    1 AXY(I+1,J)=SYMBOL
      GO TO 700
    2 AXY(I+1,J+1)=SYMBOL
      GO TO 700
    3 AXY(I,J+1)=SYMBOL
  700 CONTINUE
 701  CONTINUE
C      FIND THE BOUNDARY
      IF(KFLAG.EQ.1) GO TO 401
      DO 300 I=1,N
      DO 300 J=1,N
      IF(AXY(I,J).EQ.SYMBOL) GO TO 310
  300 CONTINUE
C
C      THE FIRST PIXEL HAS BEEN FOUND ON THE BOUNDARY
  310 XY(1,1)=I
      XY(1,2)=J
      KSTART=2
      J1=1
      KSKIP=0
```

```
C       LOOK FORTHE NEXT PIXEL ON THE BOUNDARY
  295  J1=J1+1
       B8(1)=AXY(I,J-1)
       B8(2)=AXY(I+1,J-1)
       B8(3)=AXY(I+1,J)
       B8(4)=AXY(I+1,J+1)
       B8(5)=AXY(I,J+1)
       B8(6)=AXY(I-1,J+1)
       B8(7)=AXY(I-1,J)
       B8(8)=AXY(I-1,J-1)
C
       DO 320 K=KSTART,8
       IF(K.EQ.KSKIP) GO TO 320
       IF(B8(K).EQ.SYMBOL) GO TO 325
  320  CONTINUE
       KSTOP=KSTART-1
       DO 322 K=1,KSTOP
       IF(K.EQ.KSKIP) GO TO 322
       IF(B8(K).EQ.SYMBOL) GO TO 325
  322  CONTINUE
       GO TO 350
C
  325  KSTART=K+6
       KSKIP=K+4
       IF(KSTART.GT.8) KSTART=KSTART-8
       IF(KSKIP.GT.8) KSKIP=KSKIP-8
C
        GO TO (331,332,333,334,335,336,337,338),K
 331   XY(J1,1)=I
       XY(J1,2)=J-1
       GO TO 329
 332   XY(J1,1)=I+1
       XY(J1,2)=J-1
       GO TO 329
 333   XY(J1,1)=I+1
       XY(J1,2)=J
       GO TO 329
 334   XY(J1,1)=I+1
       XY(J1,2)=J+1
       GO TO 329
 335   XY(J1,1)=I
       XY(J1,2)=J+1
       GO TO 329
 336   XY(J1,1)=I-1
       XY(J1,2)=J+1
        GO TO 329
 337   XY(J1,1)=I-1
       XY(J1,2)=J
       GO TO 329
 338   XY(J1,1)=I-1
       XY(J1,2)=J-1
```

```
C
  329   I=XY(J1,1)
        J=XY(J1,2)
        IF(J1.GT.400) GO TO 400
        IF(I.NE.XY(1,1)) GO TO 295
        IF(J.NE.XY(1,2)) GO TO 295
  400   CONTINUE
  401   CONTINUE
        DO 410 I=1,N
        DO 410 J=1,N
  410   AXY(I,J)=BLANK
        DO 420 I=1,J1
        I1=XY(I,1)
        I2=XY(I,2)
  420   AXY(I1,I2)=SYMBOL
C
  350   WRITE(6,220) J1
  220   FORMAT(2X,I5)
C
        DO 51 I=1,N
   51   WRITE(6,201) (AXY(I,J),J=1,N)
  201   FORMAT(2X,128A1)
  500   STOP
        END
/DATA
/INCLUDE FGR710XY,FGR710XZ,FGR710YZ,DMBFILE
```

Data Set 1 - xy plane

x	y	x	y	x	y	x	y	x	y	x	y
0.	-14.	53.	-2.	5.	13.	-18.	11.	-37.	-4.	-22.	-15.
1.	-14.	54.	-1.	4.	13.	-18.	10.	-37.	-5.	-23.	-16.
2.	-14.	55.	-1.	3.	14.	-17.	10.	-38.	-6.	-24.	-16.
3.	-13.	55.	0.	2.	14.	-17.	9.	-38.	-7.	-24.	-17.
4.	-13.	54.	1.	1.	15.	-18.	8.	-38.	-8.	-25.	-17.
5.	-12.	53.	2.	-1.	15.	-18.	7.	-38.	-9.	-26.	-18.
6.	-12.	52.	2.	-2.	16.	-18.	6.	-38.	-10.	-27.	-18.
7.	-12.	51.	2.	-3.	16.	-17.	6.	-38.	-11.	-27.	-19.
8.	-11.	50.	2.	-4.	16.	-18.	5.	-38.	-12.	-28.	-19.
9.	-11.	49.	2.	-5.	17.	-19.	5.	-38.	-13.	-29.	-20.
10.	-10.	48.	3.	-6.	17.	-20.	5.	-38.	-14.	-30.	-20.
11.	-10.	47.	3.	-7.	18.	-21.	5.	-38.	-15.	-30.	-21.
12.	-9.	46.	3.	-8.	18.	-22.	5.	-37.	-15.	-31.	-21.
13.	-9.	45.	4.	-9.	19.	-23.	5.	-36.	-15.	-32.	-22.
14.	-9.	44.	5.	-10.	19.	-24.	5.	-35.	-15.	-32.	-23.
15.	-8.	43.	4.	-11.	20.	-25.	5.	-34.	-15.	-32.	-24.
16.	-8.	43.	3.	-12.	20.	-26.	5.	-34.	-14.	-31.	-25.
17.	-8.	42.	3.	-13.	21.	-27.	4.	-34.	-13.	-30.	-26.
18.	-7.	41.	3.	-14.	21.	-28.	5.	-33.	-12.	-29.	-26.
19.	-7.	40.	3.	-15.	21.	-28.	6.	-32.	-10.	-28.	-26.
20.	-7.	39.	3.	-16.	22.	-29.	7.	-32.	-11.	-27.	-25.
21.	-6.	38.	3.	-17.	22.	-30.	8.	-31.	-9.	-26.	-25.
22.	-6.	37.	3.	-18.	22.	-30.	9.	-30.	-8.	-25.	-24.
23.	-6.	36.	3.	-19.	23.	-31.	10.	-30.	-7.	-24.	-24.
24.	-6.	35.	3.	-20.	23.	-32.	11.	-29.	-6.	-23.	-24.
25.	-5.	34.	3.	-21.	24.	-32.	12.	-28.	-5.	-22.	-23.
26.	-5.	33.	4.	-22.	24.	-33.	13.	-28.	-4.	-21.	-23.
27.	-5.	32.	4.	-23.	25.	-34.	14.	-27.	-4.	-20.	-22.
28.	-5.	31.	4.	-24.	25.	-34.	15.	-26.	-4.	-19.	-22.
29.	-5.	30.	4.	-25.	26.	-35.	16.	-25.	-4.	-18.	-22.
30.	-4.	29.	5.	-26.	26.	-36.	16.	-24.	-4.	-17.	-21.
31.	-4.	28.	5.	-27.	26.	-37.	16.	-23.	-4.	-16.	-21.
32.	-4.	27.	5.	-28.	27.	-38.	16.	-22.	-5.	-15.	-21.
33.	-4.	26.	5.	-29.	27.	-38.	15.	-21.	-5.	-14.	-19.
34.	-4.	25.	6.	-30.	26.	-38.	14.	-20.	-5.	-13.	-19.
35.	-3.	24.	6.	-31.	25.	-38.	13.	-19.	-5.	-12.	-18.
36.	-3.	23.	6.	-32.	23.	-38.	12.	-18.	-5.	-11.	-18.
37.	-3.	22.	6.	-31.	22.	-38.	11.	-17.	-5.	-10.	-17.
38.	-3.	21.	7.	-30.	22.	-37.	10.	-18.	-6.	-9.	-17.
39.	-3.	20.	7.	-29.	21.	-37.	9.	-19.	-6.	-8.	-17.
40.	-3.	19.	7.	-28.	20.	-37.	8.	-19.	-7.	-7.	-17.
41.	-3.	18.	8.	-27.	19.	-37.	7.	-19.	-8.	-6.	-17.
42.	-3.	17.	8.	-26.	19.	-37.	6.	-18.	-8.	-5.	-16.
43.	-3.	16.	8.	-25.	18.	-37.	5.	-17.	-8.	-4.	-16.
43.	-4.	15.	9.	-24.	17.	-37.	4.	-17.	-9.	-3.	-16.
44.	-5.	14.	9.	-23.	17.	-37.	3.	-18.	-9.	-2.	-15.
45.	-4.	13.	9.	-22.	16.	-37.	2.	-18.	-10.	-1.	-15.
46.	-3.	12.	10.	-21.	15.	-38.	2.	-18.	-11.		
47.	-3.	11.	10.	-20.	15.	-38.	1.	-17.	-12.		
48.	-3.	10.	10.	-19.	15.	-38.	0.	-18.	-12.		
49.	-2.	9.	11.	-18.	14.	-38.	-1.	-19.	-13.		
50.	-2.	8.	11.	-17.	13.	-37.	-1.	-20.	-13.		
51.	-2.	7.	12.	-17.	12.	-37.	-2.	-21.	-14.		
52.	-2.	6.	12.	-18.	12.	-37.	-3.	-21.	-15.		

Data Set 1 - yz plane

y	z	y	z	y	z	y	z
0.	-3.	4.	3.	-8.	8.	-21.	-2.
1.	-3.	3.	4.	-9.	8.	-20.	-2.
2.	-3.	3.	5.	-10.	8.	-19.	-2.
2.	-4.	2.	5.	-11.	8.	-18.	-2.
3.	-4.	1.	5.	-12.	8.	-17.	-2.
4.	-5.	1.	7.	-13.	8.	-16.	-2.
5.	-5.	2.	7.	-14.	8.	-15.	-2.
6.	-5.	3.	7.	-15.	7.	-14.	-2.
7.	-5.	4.	7.	-14.	7.	-13.	-2.
8.	-4.	5.	7.	-13.	7.	-12.	-2.
9.	-5.	6.	7.	-12.	7.	-11.	-2.
10.	-5.	7.	7.	-11.	7.	-10.	-2.
11.	-4.	8.	7.	-10.	7.	-11.	-3.
11.	-3.	9.	7.	-9.	7.	-11.	-4.
10.	-2.	10.	7.	-8.	7.	-10.	-5.
11.	-2.	11.	7.	-7.	7.	-9.	-5.
12.	-2.	12.	7.	-6.	7.	-8.	-4.
13.	-2.	13.	7.	-5.	7.	-7.	-5.
14.	-2.	14.	7.	-4.	7.	-6.	-5.
15.	-2.	15.	7.	-3.	7.	-5.	-5.
16.	-2.	14.	8.	-2.	7.	-4.	-5.
17.	-2.	13.	8.	-1.	7.	-3.	-4.
18.	-2.	12.	8.	-1.	5.	-2.	-4.
19.	-2.	11.	8.	-2.	5.	-2.	-3.
20.	-2.	10.	8.	-3.	5.	-1.	-3.
21.	-2.	9.	8.	-3.	4.		
22.	-1.	8.	8.	-4.	3.		
23.	-1.	7.	8.	-5.	3.		
24.	-1.	6.	8.	-6.	2.		
25.	-1.	5.	8.	-7.	2.		
26.	-1.	4.	8.	-8.	2.		
27.	-1.	3.	8.	-9.	2.		
26.	1.	2.	8.	-10.	2.		
25.	1.	1.	9.	-11.	2.		
24.	1.	1.	10.	-12.	2.		
23.	1.	1.	11.	-13.	2.		
22.	1.	1.	12.	-14.	2.		
21.	2.	1.	13.	-15.	2.		
20.	2.	1.	14.	-16.	2.		
19.	2.	1.	15.	-17.	2.		
18.	2.	0.	15.	-18.	2.		
17.	2.	-1.	15.	-19.	2.		
16.	2.	-1.	14.	-20.	2.		
15.	2.	-1.	13.	-21.	2.		
14.	2.	-1.	12.	-22.	1.		
13.	2.	-1.	11.	-23.	1.		
12.	2.	-1.	10.	-24.	1.		
11.	2.	-1.	9.	-25.	1.		
10.	2.	-2.	8.	-26.	1.		
9.	2.	-3.	8.	-26.	-1.		
8.	2.	-4.	8.	-25.	-1.		
7.	2.	-5.	8.	-24.	-1.		
6.	2.	-6.	8.	-23.	-1.		
5.	2.	-7.	8.	-22.	-1.		

x	z	x	z	x	z	x	z
0.	-5.	-35.	9.	7.	5.	52.	-2.
-1.	-5.	-35.	10.	8.	5.	51.	-2.
-2.	-5.	-36.	11.	9.	5.	50.	-2.
-3.	-5.	-37.	12.	10.	5.	49.	-2.
-4.	-5.	-37.	13.	11.	5.	48.	-3.
-5.	-5.	-37.	14.	12.	5.	47.	-3.
-6.	-5.	-37.	15.	13.	5.	46.	-3.
-7.	-5.	-36.	15.	14.	5.	45.	-3.
-8.	-5.	-35.	15.	15.	5.	44.	-3.
-9.	-5.	-34.	15.	16.	5.	43.	-3.
-10.	-5.	-33.	15.	17.	5.	42.	-3.
-11.	-5.	-32.	15.	18.	5.	41.	-3.
-12.	-5.	-31.	14.	19.	5.	40.	-3.
-13.	-5.	-30.	13.	20.	5.	39.	-3.
-14.	-5.	-29.	13.	21.	5.	38.	-3.
-15.	-5.	-29.	12.	22.	5.	37.	-3.
-16.	-5.	-28.	12.	23.	5.	36.	-3.
-17.	-4.	-27.	11.	24.	5.	35.	-3.
-17.	-3.	-27.	10.	25.	5.	34.	-3.
-17.	-2.	-26.	10.	26.	5.	33.	-3.
-18.	-2.	-25.	9.	27.	5.	32.	-3.
-19.	-2.	-25.	8.	28.	5.	31.	-3.
-20.	-2.	-24.	8.	29.	6.	30.	-3.
-21.	-2.	-23.	7.	30.	6.	29.	-3.
-22.	-2.	-22.	6.	31.	6.	28.	-3.
-23.	-2.	-21.	6.	32.	6.	27.	-3.
-24.	-2.	-20.	6.	33.	6.	26.	-3.
-25.	-2.	-19.	5.	34.	6.	25.	-3.
-26.	-2.	-18.	5.	35.	5.	24.	-3.
-27.	-2.	-17.	5.	36.	5.	23.	-3.
-28.	-2.	-16.	5.	37.	5.	22.	-3.
-29.	-2.	-15.	5.	38.	5.	21.	-3.
-30.	-2.	-14.	5.	39.	5.	20.	-3.
-31.	-2.	-13.	5.	40.	5.	19.	-3.
-32.	-2.	-12.	5.	41.	5.	18.	-3.
-33.	-1.	-11.	5.	42.	5.	17.	-3.
-34.	-1.	-10.	5.	43.	4.	16.	-3.
-35.	-1.	-9.	5.	44.	4.	15.	-3.
-36.	-1.	-8.	5.	45.	3.	14.	-3.
-37.	-1.	-7.	5.	46.	3.	13.	-3.
-38.	1.	-6.	5.	47.	2.	12.	-3.
-37.	2.	-5.	5.	48.	2.	11.	-3.
-36.	2.	-4.	5.	49.	2.	10.	-3.
-35.	3.	-3.	5.	50.	2.	9.	-3.
-34.	3.	-2.	5.	51.	2.	8.	-3.
-33.	3.	-1.	5.	52.	1.	8.	-4.
-34.	4.	0.	5.	53.	1.	7.	-5.
-34.	5.	1.	5.	54.	1.	6.	-5.
-35.	6.	2.	5.	55.	1.	5.	-5.
-36.	6.	3.	5.	55.	-1.	4.	-5.
-36.	7.	4.	5.	54.	-1.	3.	-5.
-35.	7.	5.	5.	53.	-1.	2.	-5.
-35.	8.	6.	5.			1.	-5.

Data Set 2 - xy plane

x	y	x	y
0.	12.	-25.	23.
1.	12.	-24.	22.
2.	12.	-23.	22.
3.	12.	-23.	21.
4.	12.	-22.	20.
5.	12.	-22.	19.
5.	13.	-21.	18.
4.	14.	-20.	17.
3.	14.	-20.	16.
2.	14.	-20.	15.
1.	15.	-19.	15.
0.	15.	-18.	15.
-1.	16.	-17.	14.
-2.	16.	-16.	13.
-3.	17.	-16.	12.
-4.	17.	-15.	12.
-5.	17.	-14.	12.
-6.	18.	-13.	12.
-7.	18.	-12.	12.
-8.	19.	-11.	12.
-9.	19.	-10.	12.
-10.	20.	-9.	12.
-11.	20.	-8.	12.
-12.	21.	-7.	12.
-13.	21.	-6.	12.
-14.	21.	-5.	12.
-15.	22.	-4.	12.
-16.	22.	-3.	12.
-17.	22.	-2.	12.
-18.	23.	-1.	12.
-19.	23.		
-20.	24.		
-21.	24.		
-22.	25.		
-23.	25.		
-24.	26.		
-25.	26.		
-26.	26.		
-27.	27.		
-28.	27.		
-28.	26.		
-27.	25.		
-26.	24.		

Data Set 2 - yz plane

y	z
12.	-2.
13.	-2.
14.	-2.
15.	-2.
16.	-2.
17.	-2.
18.	-2.
19.	-2.
20.	-2.
21.	-2.
22.	-1.
23.	-1.
24.	-1.
25.	-1.
26.	-1.
27.	-1.
26.	0.
26.	1.
25.	1.
24.	1.
23.	1.
22.	1.
21.	2.
20.	2.
19.	2.
18.	2.
17.	2.
16.	2.
15.	2.
14.	2.
13.	2.
12.	2.
12.	1.
12.	0.
12.	-1.

Data Set 2 - xz plane

x	z	x	z
0.	-2.	-24.	2.
-1.	-2.	-23.	2.
-2.	-2.	-22.	2.
-3.	-2.	-21.	2.
-4.	-2.	-20.	2.
-5.	-2.	-19.	2.
-6.	-2.	-18.	2.
-7.	-2.	-17.	2.
-8.	-2.	-16.	2.
-9.	-2.	-15.	2.
-10.	-2.	-14.	2.
-11.	-2.	-13.	2.
-12.	-2.	-12.	2.
-13.	-2.	-11.	2.
-14.	-2.	-10.	2.
-15.	-2.	-9.	2.
-16.	-2.	-8.	2.
-17.	-2.	-7.	2.
-18.	-2.	-6.	2.
-19.	-2.	-5.	2.
-20.	-2.	-4.	2.
-21.	-2.	-3.	2.
-22.	-2.	-2.	2.
-23.	-2.	-1.	2.
-24.	-2.	0.	2.
-25.	-2.	1.	2.
-26.	-2.	2.	2.
-27.	-2.	3.	1.
-28.	-2.	4.	1.
-29.	-1.	5.	1.
-30.	-1.	5.	0.
-31.	-1.	4.	-1.
-31.	0.	3.	-1.
-30.	1.	2.	-2.
-29.	1.	1.	-2.
-28.	2.		
-27.	2.		
-26.	2.		
-25.	2.		

x	y	x	y	x	y	x	y
26.	-5.	33.	4.	-21.	5.	-7.	-5.
27.	-5.	32.	4.	-22.	5.	-6.	-5.
28.	-5.	31.	4.	-23.	5.	-5.	-5.
29.	-5.	30.	4.	-24.	5.	-4.	-5.
30.	-4.	29.	5.	-25.	5.	-3.	-5.
31.	-4.	28.	5.	-26.	5.	-2.	-5.
32.	-4.	27.	5.	-27.	4.	-1.	-5.
33.	-4.	26.	5.	-28.	5.	0.	-5.
34.	-4.	25.	5.	-29.	5.	1.	-5.
35.	-3.	24.	5.	-30.	5.	2.	-5.
36.	-3.	23.	5.	-31.	5.	3.	-5.
37.	-3.	22.	5.	-32.	5.	4.	-5.
38.	-3.	21.	5.	-33.	5.	5.	-5.
39.	-3.	20.	5.	-34.	5.	6.	-5.
40.	-3.	19.	5.	-35.	5.	7.	-5.
41.	-3.	18.	5.	-36.	5.	8.	-5.
42.	-3.	17.	5.	-36.	4.	9.	-5.
43.	-3.	16.	5.	-36.	3.	10.	-5.
43.	-4.	15.	5.	-36.	2.	11.	-5.
44.	-5.	14.	5.	-36.	1.	12.	-5.
45.	-4.	13.	5.	-36.	0.	13.	-5.
46.	-3.	12.	5.	-36.	-1.	14.	-5.
47.	-3.	11.	5.	-36.	-2.	15.	-5.
48.	-3.	10.	5.	-36.	-3.	16.	-5.
49.	-2.	9.	5.	-36.	-4.	17.	-5.
50.	-2.	8.	5.	-36.	-5.	18.	-5.
51.	-2.	7.	5.	-35.	-5.	19.	-5.
52.	-2.	6.	5.	-34.	-5.	20.	-5.
53.	-2.	5.	5.	-33.	-5.	21.	-5.
54.	-1.	4.	5.	-32.	-5.	22.	-5.
55.	-1.	3.	5.	-31.	-5.	23.	-5.
55.	0.	2.	5.	-30.	-5.	24.	-5.
54.	1.	1.	5.	-29.	-5.	25.	-5.
53.	2.	0.	5.	-28.	-5.		
52.	2.	-1.	5.	-27.	-5.		
51.	2.	-2.	5.	-26.	-5.		
50.	2.	-3.	5.	-25.	-5.		
49.	2.	-4.	5.	-24.	-5.		
48.	3.	-5.	5.	-23.	-5.		
47.	3.	-6.	5.	-22.	-5.		
46.	3.	-7.	5.	-21.	-5.		
45.	4.	-8.	5.	-20.	-5.		
44.	5.	-9.	5.	-19.	-5.		
43.	4.	-10.	5.	-18.	-5.		
43.	3.	-11.	5.	-17.	-5.		
42.	3.	-12.	5.	-16.	-5.		
41.	3.	-13.	5.	-15.	-5.		
40.	3.	-14.	5.	-14.	-5.		
39.	3.	-15.	5.	-13.	-5.		
38.	3.	-16.	5.	-12.	-5.		
37.	3.	-17.	5.	-11.	-5.		
36.	3.	-18.	5.	-10.	-5.		
35.	3.	-19.	5.	-9.	-5.		
34.	3.	-20.	5.	-8.	-5.		

Data Set 3 - yz plane

y	z
0.	-3.
1.	-3.
2.	-3.
3.	-2.
4.	-1.
4.	0.
4.	1.
3.	2.
2.	3.
2.	4.
1.	5.
0.	5.
-1.	5.
-2.	4.
-2.	3.
-3.	2.
-4.	1.
-4.	0.
-4.	-1.
-3.	-2.
-2.	-3.
-1.	-3.

x	z	x	z	x	z	x	z
0.	-3.	-25.	5.	29.	6.	28.	-3.
-1.	-3.	-24.	5.	30.	6.	27.	-3.
-2.	-3.	-23.	5.	31.	6.	26.	-3.
-3.	-3.	-22.	5.	32.	6.	25.	-3.
-4.	-3.	-21.	5.	33.	6.	24.	-3.
-5.	-3.	-20.	5.	34.	6.	23.	-3.
-6.	-3.	-19.	5.	35.	5.	22.	-3.
-7.	-3.	-18.	5.	36.	5.	21.	-3.
-8.	-3.	-17.	5.	37.	5.	20.	-3.
-9.	-3.	-16.	5.	38.	5.	19.	-3.
-10.	-3.	-15.	5.	39.	5.	18.	-3.
-11.	-3.	-14.	5.	40.	5.	17.	-3.
-12.	-3.	-13.	5.	41.	5.	16.	-3.
-13.	-3.	-12.	5.	42.	5.	15.	-3.
-14.	-3.	-11.	5.	43.	4.	14.	-3.
-15.	-3.	-10.	5.	44.	4.	13.	-3.
-16.	-3.	-9.	5.	45.	3.	12.	-3.
-17.	-2.	-8.	5.	46.	3.	11.	-3.
-18.	-2.	-7.	5.	47.	2.	10.	-3.
-19.	-2.	-6.	5.	48.	2.	9.	-3.
-20.	-2.	-5.	5.	49.	2.	8.	-3.
-21.	-2.	-4.	5.	50.	2.	7.	-3.
-22.	-2.	-3.	5.	51.	2.	6.	-3.
-23.	-2.	-2.	5.	52.	1.	5.	-3.
-24.	-2.	-1.	5.	53.	1.	4.	-3.
-25.	-2.	0.	5.	54.	1.	3.	-3.
-26.	-2.	1.	5.	55.	1.	2.	-3.
-27.	-2.	2.	5.	55.	-1.	1.	-3.
-28.	-2.	3.	5.	54.	-1.		
-29.	-2.	4.	5.	53.	-1.		
-30.	-2.	5.	5.	52.	-2.		
-31.	-2.	6.	5.	51.	-2.		
-32.	-2.	7.	5.	50.	-2.		
-33.	-1.	8.	5.	49.	-2.		
-34.	-1.	9.	5.	48.	-3.		
-35.	-1.	10.	5.	47.	-3.		
-36.	-1.	11.	5.	46.	-3.		
-37.	-1.	12.	5.	45.	-3.		
-38.	1.	13.	5.	44.	-3.		
-37.	2.	14.	5.	43.	-3.		
-36.	2.	15.	5.	42.	-3.		
-35.	3.	16.	5.	41.	-3.		
-34.	3.	17.	5.	40.	-3.		
-33.	3.	18.	5.	39.	-3.		
-34.	4.	19.	5.	38.	-3.		
-34.	5.	20.	5.	37.	-3.		
-33.	5.	21.	5.	36.	-3.		
-32.	5.	22.	5.	35.	-3.		
-31.	5.	23.	5.	34.	-3.		
-30.	5.	24.	5.	33.	-3.		
-29.	5.	25.	5.	32.	-3.		
-28.	5.	26.	5.	31.	-3.		
-27.	5.	27.	5.	30.	-3.		
-26.	5.	28.	5.	29.	-3.		

Data Set 4 - xy plane

x	y	x	y
-18.	5.	-36.	-6.
-19.	5.	-36.	-7.
-20.	5.	-36.	-8.
-21.	5.	-36.	-9.
-22.	5.	-36.	-10.
-23.	5.	-36.	-11.
-24.	5.	-36.	-12.
-25.	5.	-36.	-13.
-26.	5.	-36.	-14.
-27.	5.	-36.	-15.
-28.	5.	-35.	-15.
-28.	6.	-34.	-15.
-29.	7.	-34.	-14.
-30.	8.	-33.	-13.
-30.	9.	-32.	-12.
-31.	10.	-32.	-11.
-32.	11.	-31.	-10.
-32.	12.	-30.	-9.
-33.	13.	-30.	-8.
-34.	14.	-29.	-7.
-34.	15.	-28.	-6.
-35.	15.	-28.	-5.
-36.	15.	-27.	-4.
-36.	14.	-26.	-5.
-36.	13.	-25.	-5.
-36.	12.	-24.	-5.
-36.	11.	-23.	-5.
-36.	10.	-22.	-5.
-36.	9.	-21.	-5.
-36.	8.	-20.	-5.
-36.	7.	-19.	-5.
-36.	6.	-18.	-5.
-36.	5.	-18.	-4.
-36.	4.	-18.	-3.
-36.	3.	-18.	-2.
-36.	2.	-18.	-1.
-36.	1.	-18.	0.
-36.	0.	-18.	1.
-36.	-1.	-18.	2.
-36.	-2.	-18.	3.
-36.	-3.	-18.	4.
-36.	-4.		
-36.	-5.		

y	z	y	z
1.	4.	-1.	13.
1.	5.	-1.	12.
1.	6.	-1.	11.
2.	7.	-1.	10.
3.	7.	-1.	9.
4.	7.	-2.	8.
5.	7.	-3.	8.
6.	7.	-4.	8.
7.	7.	-5.	8.
8.	7.	-6.	8.
9.	7.	-7.	8.
10.	7.	-8.	8.
11.	7.	-9.	8.
12.	7.	-10.	8.
13.	7.	-11.	8.
14.	7.	-12.	8.
15.	7.	-13.	8.
14.	8.	-14.	8.
13.	8.	-15.	7.
12.	8.	-14.	7.
11.	8.	-13.	7.
10.	8.	-12.	7.
9.	8.	-11.	7.
8.	8.	-10.	7.
7.	8.	-9.	7.
6.	8.	-8.	7.
5.	8.	-7.	7.
4.	8.	-6.	7.
3.	8.	-5.	7.
2.	8.	-4.	7.
1.	9.	-3.	7.
1.	10.	-2.	7.
1.	11.	-1.	6.
1.	12.	-1.	5.
1.	13.	-1.	4.
1.	14.	0.	4.
0.	14.		

x	z
-18.	4.
-19.	4.
-20.	4.
-21.	4.
-22.	4.
-23.	4.
-24.	4.
-25.	4.
-26.	4.
-27.	4.
-28.	4.
-29.	4.
-30.	4.
-31.	4.
-32.	4.
-33.	4.
-34.	5.
-35.	5.
-35.	6.
-34.	6.
-33.	6.
-34.	7.
-34.	8.
-34.	9.
-35.	10.
-35.	11.
-35.	12.
-36.	12.
-36.	13.
-36.	14.
-35.	14.
-34.	14.
-33.	14.
-32.	14.
-31.	14.
-30.	13.
-29.	12.
-28.	12.
-28.	11.
-27.	11.
-26.	10.
-26.	9.
-25.	9.
-24.	8.
-24.	7.
-23.	7.
-22.	6.
-21.	5.
-20.	5.
-19.	5.
-18.	5.